

# Statics

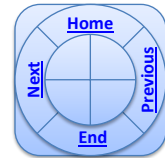


## Chapter Four

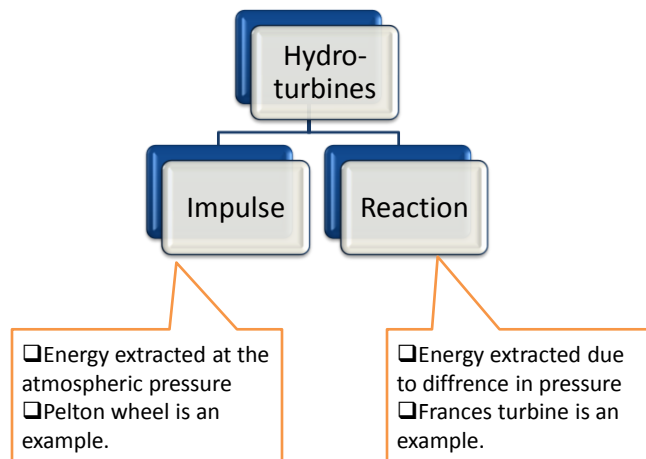
### Impulse turbines-Pelton wheel

By

Laith Batarseh

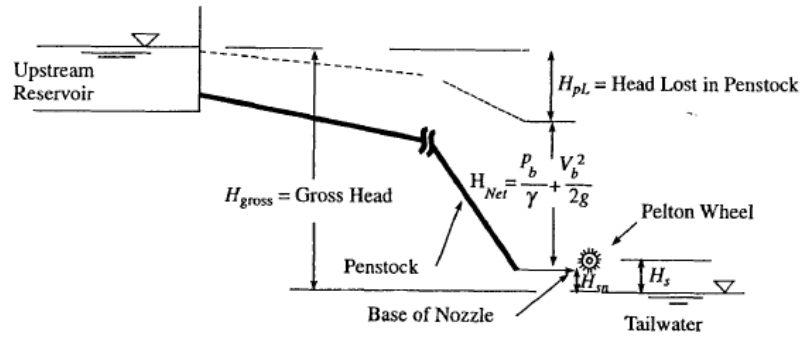


### Impulse turbines-Pelton wheel



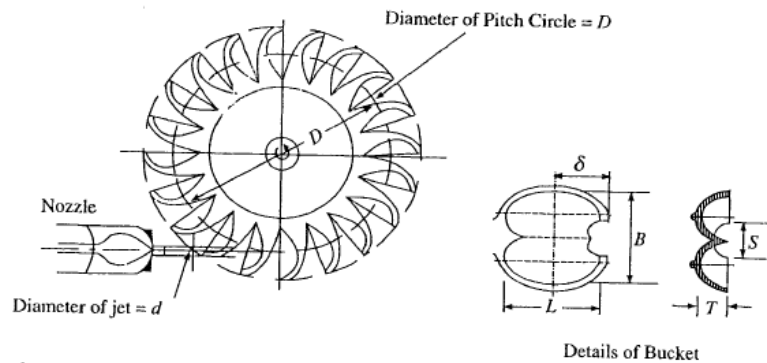
## Impulse turbines-Pelton wheel

### System description



## Impulse turbines-Pelton wheel

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## Impulse turbines-Pelton wheel

### Theoretical analysis

$$\text{net head} = H = H_{\text{gross}} - H_{\text{pL}} - H_{\text{sn}}$$

where  $H_{\text{gross}}$  = Gross head = Difference in water surface elevation of upstream reservoir and tailwater level.

$H_{\text{pL}}$  = Head loss in the penstock

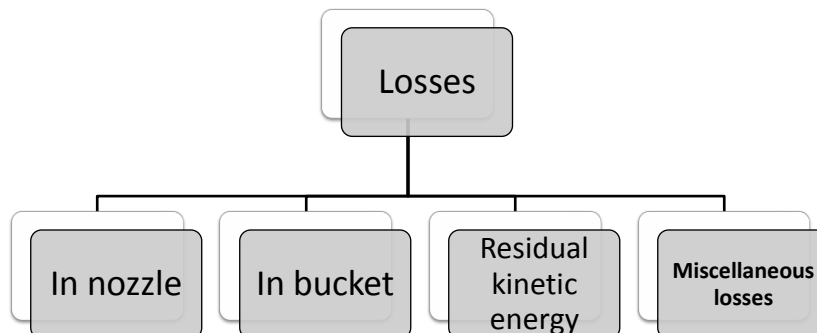
$H_{\text{sn}}$  = Height of the lowest nozzle above the tailwater level

Also, 
$$H = \frac{p_b}{\gamma} + \frac{V_b^2}{2g}$$

where  $\frac{p_b}{\gamma}$  = Pressure head at the base of the nozzle and  $\frac{V_b^2}{2g}$  = Velocity head at the base of the nozzle.

## Impulse turbines-Pelton wheel

### Kinetic energy of jet



## Impulse turbines-Pelton wheel

### Losses in nozzle

$$H_{Ln} = \left( \frac{1}{C_v^2} - 1 \right) \left( 1 - \left( \frac{A_1}{A_b} \right)^2 \right) \frac{V_1^2}{2g}$$

where  $A_b$  = Cross-sectional area at the base of the nozzle, and  
 $A_1$  = Cross-sectional area of the jet.  
 $C_v$  is the velocity coefficient of the nozzle

### Losses in bucket

$$H_{Lb} = \frac{v_{r1}^2}{2g} - \frac{v_{r2}^2}{2g} = (1 - K^2) \frac{v_{r1}^2}{2g} \quad K = \frac{v_{r2}}{v_{r1}} \cdot \text{bucket friction coefficient.}$$

## Impulse turbines-Pelton wheel

### Residual kinetic energy losses, $H_{Le}$

$$H_{Le} = \frac{V_2^2}{2g} \quad V_2 \text{ is the absolute velocity of water leaving the bucket.}$$

### Net head, H

Net head ( $H$ ) = Energy transmitted to the turbine ( $H_e$ ) + Losses in [nozzle ( $H_{Ln}$ ) + buckets ( $H_{Lb}$ )] + Energy going waste to tailwater ( $H_{Le}$ )

$$\begin{aligned} H &= H_e + [H_{Ln} + H_{Lb} + H_{Le}] \\ &= H_e + H_{Lke} \end{aligned}$$

where  $H_{Lke}$  = Total loss of kinetic energy in the nozzle - bucket system

## Impulse turbines-Pelton wheel

### Overall power, P

The *shaft power*  $P$  developed by the interaction of a jet with the buckets in the Pelton wheel is given by

$$P = \eta_0 \gamma QH$$

where  $\eta_0$  is the overall efficiency,  $H$  = Net head, and  $Q$  = Discharge through the nozzle.

## Impulse turbines-Pelton wheel

### Speed Ratio, $K_u$

$$\text{Speed ratio} = K_u = \frac{u}{\sqrt{2gH}}$$

$$0.43 < K_u < 0.47$$

### Specific speed, $N_s$

Peripheral speed of the wheel in rpm

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

Brake power in kW developed per jet

Net head in metres,

$$8 < N_s < 30$$

## Impulse turbines-Pelton wheel

### Specific speed, $N_s$

**Table 4.1** Specific speeds of different kinds of turbines

Type of turbine	Specific-speed range ( $N_s$ in kW- rpm-m units)
Pelton (Single jet or for each jet in a multijet unit; maximum of six jets)	8–30 ( on power per jet basis)
Francis	40–450
Kaplan	300–900

### Rotational speed, $N$

$$u = \frac{\pi DN}{60} \text{ and hence}$$

$$N = \frac{60u}{\pi D} = \frac{60 \times K_u \sqrt{2gH}}{\pi} \times \frac{1}{D}$$

## Impulse turbines-Pelton wheel

### Specific Speed in Terms of Defined Ratios

Specific speed  $N_s = \frac{N\sqrt{P}}{H^{5/4}}$  where  $P$  is the power per jet.

$$\text{Speed of rotation } N = \frac{60 \times u}{\pi D} = \frac{60 \times K_u \sqrt{2gH}}{\pi D} \quad (4.15)$$

$$\begin{aligned} \text{Considering the power per jet, } P &= \eta_0 \gamma QH = \eta_0 \gamma \times \left( \frac{\pi d^2}{4} \right) \times V_1 H \\ &= \eta_0 \gamma \times \left( \frac{\pi d^2}{4} \right) \times H \times C_v \sqrt{2gH} \end{aligned} \quad (4.16)$$

Substituting Eq. (4.15) and Eq.(4.16) in the expression for the specific speed,

$$\begin{aligned} N_s &= \frac{N\sqrt{P}}{H^{5/4}} = \frac{1}{H^{5/4}} \times \frac{60 \times K_u \sqrt{2gH}}{\pi D} \times \sqrt{\left[ \eta_0 \gamma \left( \frac{\pi}{4} \right) d^2 \times H \times C_v \sqrt{2gH} \right]} \\ N_s &= \frac{60 \times (\gamma)^{1/2} \left( \frac{\pi}{4} \right)^{1/2}}{\pi} \times K_u \sqrt{C_v} \times \left( \frac{d}{D} \right) \times (2g)^{3/4} \times (\eta_0)^{1/2} \\ N_s &= \frac{60}{\sqrt{\pi}} \times \frac{1}{2} \times (\gamma)^{1/2} (\eta_0)^{1/2} (2g)^{3/4} K_u \sqrt{C_v} \times \left( \frac{d}{D} \right) \end{aligned} \quad (4.17)$$

Substituting  $\gamma = 9.79 \text{ m}^3/\text{s}$  and  $g = 9.81 \text{ m/s}^2$ ,

$$N_s = 493.7 K_u \left( \frac{d}{D} \right) \sqrt{\eta_0 C_v} \text{ This could be written as } N_s = \frac{K_1}{\left( \frac{D}{d} \right)}$$

## Impulse turbines-Pelton wheel

### Coefficient of Velocity of the Nozzle, $C_v$

$$C_v = \frac{V_1}{\sqrt{2gH}} \quad \longrightarrow \quad Q = \left(\frac{\pi}{4}d^2\right)V_1 = \left(\frac{\pi}{4}d^2\right)C_v\sqrt{2gH}$$

$$V_1 = C_v\sqrt{2gH}$$

$$0.98 < C_v < 0.99$$

Also

$$\frac{K_u}{C_v} = \frac{u}{V_1}$$

### Jet Ratio, $m$

$$\text{Jet ratio} = m = \frac{D}{d} \begin{array}{l} \text{pitch diameter of the runner} \\ \text{diameter of the jet} \end{array}$$

$$7 < m < 26$$

## Impulse turbines-Pelton wheel

### 4.2.2 Basic theory of Pelton Wheel

#### 1. Notations

The following notations are used in the derivation of the basic equations for power and efficiency of a Pelton turbine. Some of the notations are explained once again, for purpose of clarity, in the course of the derivations that follow.

$u$  = Peripheral velocity of the wheel

$V_1$  = Jet velocity = absolute velocity of jet entering the bucket

$v_{r1}$  = Relative velocity of jet at the inlet =  $(V_1 - u)$

$D$  = Pitch diameter of the wheel

$d$  = Diameter of the free jet

$\beta$  = Bucket angle = Deflection angle of the relative velocity of the jet

$\beta'$  =  $(180 - \beta)$  = Supplementary angle of bucket angle  $\beta$ .

$K$  = Bucket friction coefficient

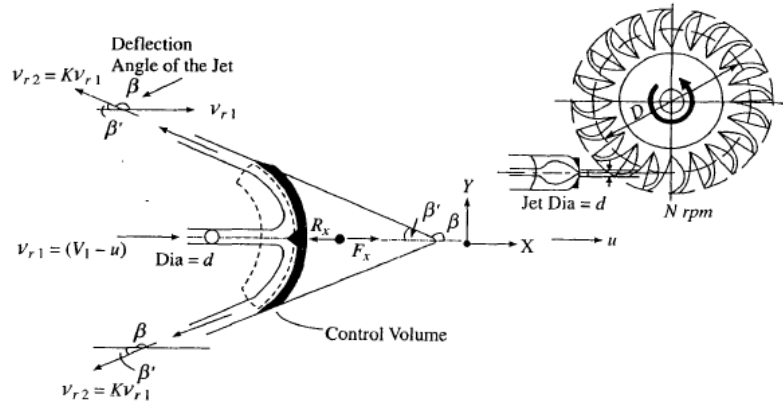
$N$  = Speed of rotation

$Q$  = Discharge carried by the jet

$H$  = Net head on the turbine

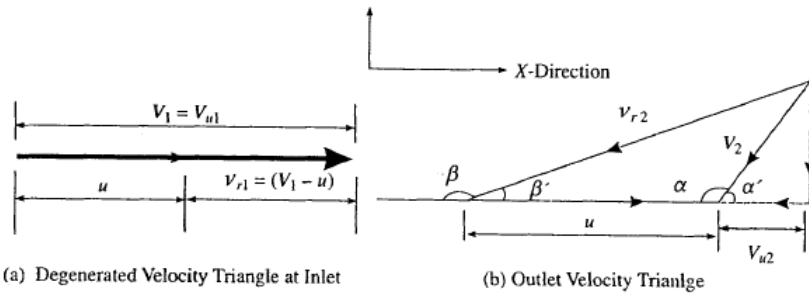
**Impulse turbines-Pelton wheel**

**4.2.2 Basic theory of Pelton Wheel**



**Impulse turbines-Pelton wheel**

**Velocity triangles**



**Fig. 4.4** Velocity triangle at the inlet and outlet of a Pelton turbine



## Impulse turbines-Pelton wheel

### Governing equations

**Force**  $F_x = \rho Q [(v_{r1}) - (-Kv_{r1} \cos \beta')]$

$$F_x = \rho Q [(V_1 - u) (1 + K \cos \beta')]$$

**Torque**  $T = F_x \frac{D}{2} = \rho Q [(V_1 - u)(1 + K \cos \beta')] \frac{D}{2}$

**Power**  $P = F_x u = \rho Q u [(V_1 - u) (1 + K \cos \beta')]$

**Head**  $H_e = \frac{P}{\gamma H} = \frac{1}{g} u [(V_1 - u) (1 + K \cos \beta')]$

**Hydraulic efficiency**  $\eta_h = \frac{H_e}{H} = \frac{u [(V_1 - u)(1 + K \cos \beta')]}{gH}$

## Impulse turbines-Pelton wheel

### Other equations

$$H_e = \frac{(V_{u1} + V_{u2})u}{g} = \frac{[V_1 + K(V_1 - u)\cos \beta' - u]u}{g}$$

$$H_e = \frac{u[(V_1 - u)(1 + K \cos \beta')]}{g}$$

$$\eta_h = \frac{H_e}{H} = \frac{u[(V_1 - u)(1 + K \cos \beta')]}{gH}$$

## Impulse turbines-Pelton wheel

### Analysis of Power, Torque and Efficiency

#### 1. Power

If  $\eta_o$ ,  $\eta_h$ ,  $\eta_m$  and  $\eta_v$  are the overall, hydraulic, mechanical and volumetric efficiencies of the turbine then they are related as

$$\eta_o = \eta_h \eta_m \eta_v = \frac{P}{\gamma Q H}$$

$$\eta_o = \frac{u[(V_1 - u)(1 + K \cos \beta')]}{gH} = \frac{\eta_o}{\eta_m \eta_v} = \frac{P}{\eta_m \eta_v \gamma Q H}$$

$$P = \eta_m \eta_v \rho Q u (V_1 - u)(1 + K \cos \beta')$$

$$P = C_1 u (V_1 - u) = P = C_2 \frac{u}{V_1} \left(1 - \frac{u}{V_1}\right)$$

$C_1$  and  $C_2$  are constants for a given turbine unit

## Impulse turbines-Pelton wheel

$\epsilon = u/V_1$ . The power is given by

$$P = C_2 \epsilon (1 - \epsilon)$$

maximum power  $\frac{dP}{d\epsilon} = 0$        $\frac{dP}{d\epsilon} = C_2 (1 - \epsilon) = 0$

This gives the condition for maximum power as  $\epsilon = 0.5$ , that is  $u = \frac{V_1}{2}$ .

Substituting  $u = K_u \sqrt{2gH}$  and  $V_1 = C_v \sqrt{2gH}$ , the condition of maximum power reduces to

$$K_u = \frac{C_v}{2} \quad (4.28)$$

## Impulse turbines-Pelton wheel

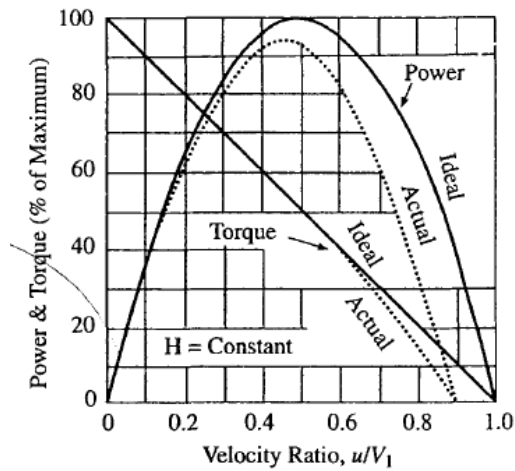


Fig. 4.5 Variation of power and torque with speed ratio

## Impulse turbines-Pelton wheel

### 2. Torque

$$T = \frac{P}{\omega}$$

Since  $u = \frac{\pi DN}{60}$  and angular velocity  $\omega = \frac{2\pi N}{60}$  radians/second,  $u = \frac{\omega D}{2}$ .

$$P = \eta_m \eta_v \rho Q u (V_1 - u)(1 + K \cos \beta')$$

$$T = \frac{P}{\omega} = \frac{\eta_m \eta_v \rho Q D}{2} (V_1 - u)(1 + K \cos \beta')$$

Also, in terms of  $H_e$ ,

$$\begin{aligned} T &= \frac{\gamma Q H_e}{\omega} = \gamma Q H_e \left( \frac{D}{2u} \right) \\ &= C_4 (V_1 - u) \end{aligned}$$

## Impulse turbines-Pelton wheel

### Runaway Speed

$$P = \eta_m \eta_v \rho Q u (V_1 - u)(1 + K \cos \beta')$$

the power  $P = 0$  when  $u = 0$  and when  $u = V_1$

$$u = V_1, \text{ the wheel is running at maximum velocity of } u = \frac{\pi DN}{60} = V_1$$

normal wheel velocity be  $u_0$

$$\text{Under normal conditions } u_0 = K_u \sqrt{2gH} \text{ and } V_1 = C_v \sqrt{2gH}.$$

Under runaway conditions, the theoretical runaway speed is

$$u_R = V_1 = C_v \sqrt{2gH} = \left( \frac{C_v}{K_u} \right) u_0$$

## Impulse turbines-Pelton wheel

Since  $u \propto N$ , theoretical runaway speed in rpm is

$$N_R = \left( \frac{C_v}{K_u} \right) N_0$$

where  $N_0 =$  Normal operative speed.

### 3. Wheel Efficiency, $\eta_w$

Wheel efficiency =  $\eta_w = \frac{\text{Power transmitted to the wheel by the water jet}}{\text{Power input to the wheel as kinetic energy}}$

$$\eta_w = \frac{\rho Q u [(V_1 - u)(1 + K \cos \beta')]}{\rho Q \frac{V_1^2}{2}} = 2 \left( \frac{u}{V_1} \right) \left( 1 - \frac{u}{V_1} \right) (1 + K \cos \beta')$$

## Impulse turbines-Pelton wheel

the wheel efficiency does not include the losses in the nozzle

$$\eta_w = \frac{\eta_h}{(C_v^2)}$$

Substituting  $u = \frac{V_1}{2}$  in the expression for  $\eta_w$  given by Eq. 4.33, the value of maximum wheel efficiency is obtained as

$$\eta_{w\max} = 2 \times \frac{1}{2} \times \frac{1}{2} \times (1 + K \cos \beta')$$

$$\eta_{w\max} = \frac{1}{2} (1 + K \cos \beta') \quad (4.34)$$

## Impulse turbines-Pelton wheel

### Nozzle Efficiency

$$\text{Nozzle efficiency} = \eta_n = \frac{\text{Energy at nozzle outlet}}{\text{Energy at nozzle inlet}}$$

$$\eta_n = \frac{\left(\frac{V_1^2}{2g}\right)}{H} = \frac{V_1^2}{2gH} = \frac{(C_v^2 \times 2gH)}{2gH} = C_v^2$$

$$\eta_h = \eta_w \cdot \eta_n$$

## Impulse turbines-Pelton wheel

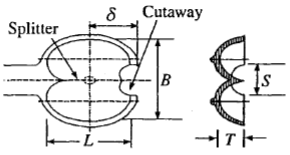
### COMPONENTS OF PELTON TURBINE

#### 1. Wheel

#### 2. Buckets

$$Z = \frac{m}{2} + 15$$

**Table 4.2** Basic geometrical dimensions of the Pelton bucket

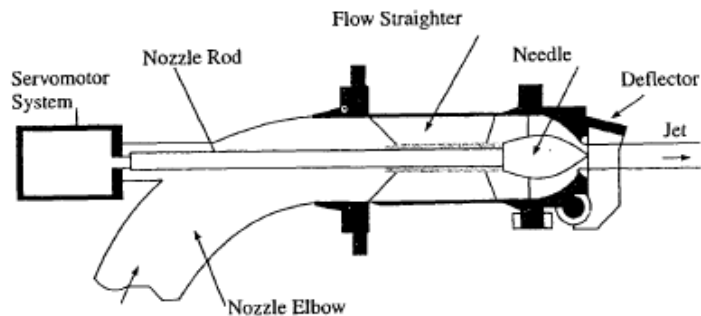
						
Radial length = $L$	Axial width = $B$	Depth of bucket = $T$	Axial width of cutaway = $S$	Radial length of cutaway = $\delta$	Inlet bucket angle = $\beta_1$	Outlet bucket angle = $\beta_2$
$L/d$	$B/d$	$T/d$	$S/d$	$\delta/d$	$\beta_1$	$\beta_2$
2.0 to 3.0	3.0 to 5.0	0.8 to 1.2	1.1 to 1.2	0.18 to 0.20	$5^\circ$ to $8^\circ$	$165^\circ$ to $170^\circ$

## Impulse turbines-Pelton wheel

### COMPONENTS OF PELTON TURBINE

#### Nozzle with Control Arrangement

#### 1. Nozzle



**Fig. 4.6** Typical nozzle assembly of a Pelton turbine

## Impulse turbines-Pelton wheel

### COMPONENTS OF PELTON TURBINE

#### Nozzle with Control Arrangement

#### 2. Deflectors

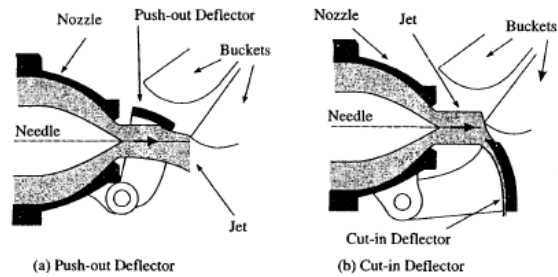


Fig. 4.7 Deflectors: (a) Push-out deflector (b) Cut-in deflector

## Impulse turbines-Pelton wheel

### COMPONENTS OF PELTON TURBINE

#### Manifold, Braking Jet and Auxiliary Jet

#### 1. Manifold

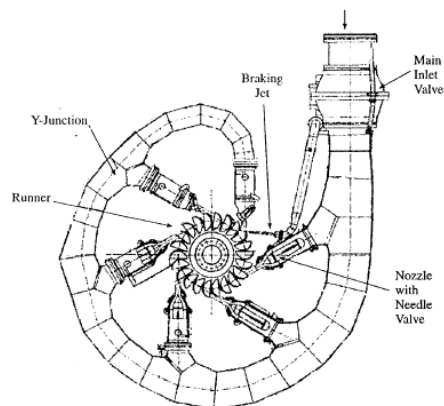


Fig. 4.8 Schematic sketch of the manifold and jet arrangements in a six-jet Pelton turbine

## Impulse turbines-Pelton wheel

### COMPONENTS OF PELTON TURBINE

**Manifold, Braking Jet and Auxiliary Jet**

**2. Braking Jet**

**3. Auxiliary Nozzle (Relief Nozzle)**

**Enclosing Chamber (Casing)**

## Impulse turbines-Pelton wheel

### Working Proportions of Pelton Turbine

**Table 4.3** Salient working proportions and ranges of important parameters of a Pelton turbine

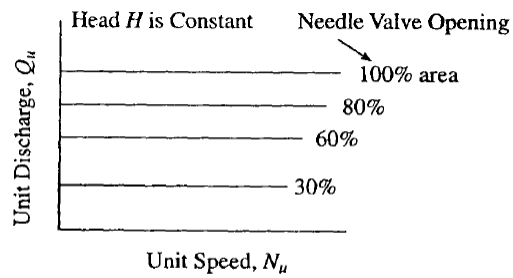
S.No	Item	Range	Normal value
1	Head, $H$	100–1870 m	
2	Speed	75–1000 rpm	
3	Maximum capacity	420 MW	
4	Speed ratio, $K_w$	0.43–0.47	0.46
5	Coefficient of velocity, $C_v = \frac{V_1}{\sqrt{2gH}}$	0.98 to 0.99 for full valve opening	0.985
6	Jet ratio = $m = \frac{D}{d}$	$N_s = 209 \left( \frac{d}{D} \right)$	7 to 26
7	Runaway speed, $N_R$	1.85 $N$ – 1.90 $N$	1.87 $N$
8	Bucket angle, $\beta_2$	170–165°	165°
9	Number of buckets	$Z = \frac{m}{2} + 15$	
10	Specific Speed, $N_s$	Single jet: $N_s = 8–30$ Also per jet in multijet Pelton. (Max. of 6 jets)	



## Impulse turbines-Pelton wheel

### 4.4 PERFORMANCE CHARACTERISTICS OF PELTON TURBINES

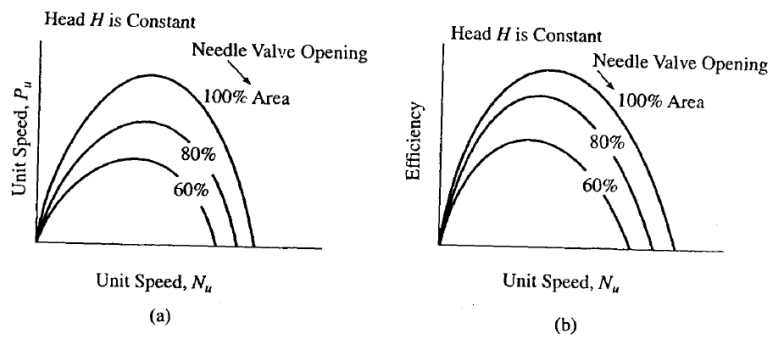
#### 1. Variation of Discharge



**Fig. 4.9** Variation of discharge with speed in a Pelton turbine

## Impulse turbines-Pelton wheel

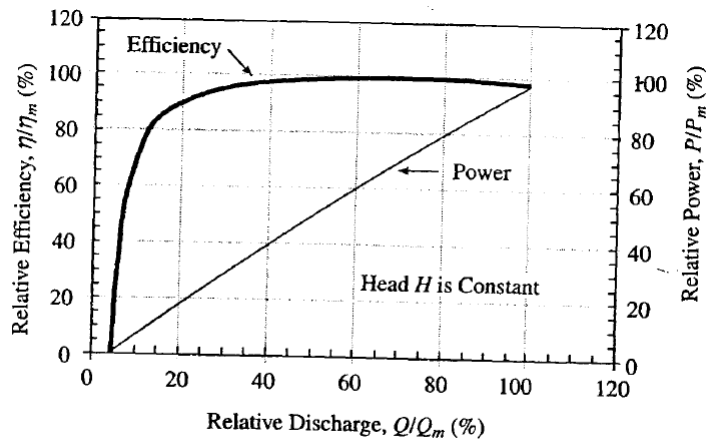
#### 2. Variation of Power



**Fig. 4.10** Variation of (a) power, and (b) efficiency with speed in a Pelton turbine

## Impulse turbines-Pelton wheel

### 3. Variation of Efficiency



**Fig. 4.11** Variation of power and efficiency with discharge in a Pelton turbine

## Impulse turbines-Pelton wheel

### 4.9 SELECTION OF TURBINE TYPE

- (a) **Gross Head and Net Head** Magnitude and variation in a water year
- (b) **Available Discharge** Average daily/ weekly magnitude and probabilities of occurrence in a water year
- (c) **Power Potential** Daily/weekly over a year

## Impulse turbines-Pelton wheel

### 4.9 SELECTION OF TURBINE TYPE

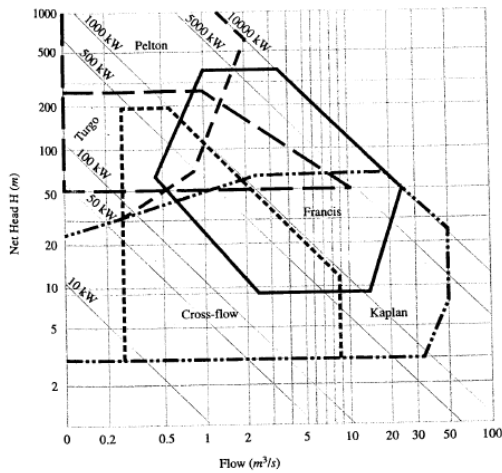


Fig. 4.16 Turbine selection chart for small hydro projects (SHP)

## Impulse turbines-Pelton wheel

### 4.9 SELECTION OF TURBINE TYPE

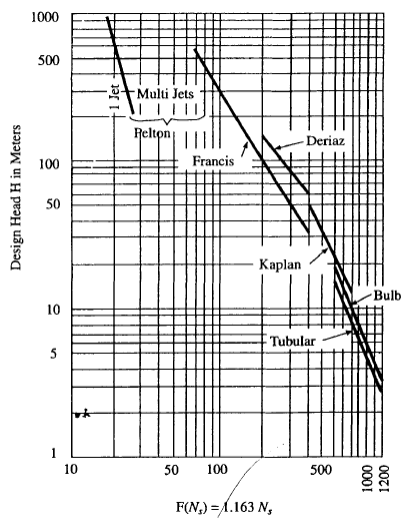


Fig. 4.17 Chart for selection of turbine based on specific speed

## Impulse turbines-Pelton wheel

### \*EXAMPLE 4.1

The water jet in a Pelton wheel is 8 cm in diameter and has a velocity of 93 m/s. The rotational speed of the wheel is 600 rpm and the deflection angle of the jet at the bucket is  $170^\circ$ . If the ratio of bucket velocity to jet velocity is 0.47, determine the (a) diameter of the wheel, and (b) power transferred to the wheel by the jet. Take bucket friction coefficient  $K = 0.96$ .

#### Solution

Given:  $d = 0.08$  m,  $V_1 = 93$  m/s,  $N = 600$  rpm,  $\beta_2 = 170^\circ$ ,  $\frac{u}{V_1} = 0.47$ ,  $K = 0.96$   
 Since,  $u/V_1 = 0.47$ ,

$$u = 0.47 \times 93 = 43.71 \text{ m/s}$$

$$(a) u = \frac{\pi DN}{60}$$

$$\text{Diameter of the wheel } D = \frac{60 \times u}{\pi N} = \frac{60 \times 43.71}{\pi \times 600} = 1.391 \text{ m}$$

$$\beta'_2 = 180 - 170 = 10^\circ$$

$$\text{Discharge } Q = \frac{\pi}{4} (0.08)^2 \times 93 = 0.4675 \text{ m}^3/\text{s}$$

(b) Power transmitted to the runner by the jet = Theoretical power =

$$P = \rho Q u (V_1 - u) (1 + K \cos \beta'_2)$$

$$P = 0.998 \times 0.4675 \times 43.71 \times (93 - 43.71) (1 + 0.96 \times \cos 10^\circ)$$

$$P = 1955 \text{ kW}$$

## Impulse turbines-Pelton wheel

### \*EXAMPLE 4.3

The following data pertains to a Pelton turbine:

Speed ratio = 0.46	Coefficient of velocity of the nozzle = 0.97
Mechanical efficiency = 0.92	Speed = 300 rpm
Pitch diameter of runner = 2.0 m	Bucket friction coefficient $K = 0.97$
Jet deflection angle at bucket = $170^\circ$	Discharge = $0.5 \text{ m}^3/\text{s}$

Calculate the (a) power transmitted by the jet to the wheel, and (b) net head.

#### Solution

Power transmitted by the jet to the wheel (Euler power) is given by

$$P = \rho Q u (V_1 - u) (1 + K \cos \beta'_2)$$

$$u = \frac{\pi DN}{60} = \frac{\pi \times 2.0 \times 300}{60} = 31.423 \text{ m/s}$$

$$\frac{u}{V_1} = \frac{K_u}{C_v} \text{ and hence } V_1 = \frac{uC_v}{K_u} = \frac{31.426 \times 0.97}{0.46} = 66.26 \text{ m/s}$$

$$P = 0.998 \times 0.5 \times 31.423 \times (66.26 - 31.423) (1 + 0.97 \cos 10^\circ)$$

$$= 15.68 \times 34.84 \times 1.9553 = 1068 \text{ kW}$$

$$\text{Since, } u = K_u \sqrt{2gH}$$

$$\text{Net head } H = \frac{u^2}{2gK_u^2} = \frac{(31.423)^2}{2 \times 9.81 \times (0.46)^2} = 237.8 \text{ m}$$

## Impulse turbines-Pelton wheel

### \*\*EXAMPLE 4.7

A Pelton wheel of 2.5 m diameter operates under the following conditions:

Net available head = 300 m	Jet deflection angle in the bucket = 165°
Coefficient velocity of the jet = 0.98	Diameter of jet = 20 cm
Bucket friction coefficient = 0.95	Mechanical efficiency = 0.95
Speed = 300 rpm	

Determine (a) the shaft power (b) hydraulic efficiency, and (c) specific speed.

### Solution

$$\beta'_2 = 180 - 165 = 15^\circ$$

$$\text{Velocity of jet } V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 300} = 75.186 \text{ m/s}$$

$$\text{Discharge } Q = A_1 V_1 = \frac{\pi}{4} (0.2)^2 \times 75.186 = 2.362 \text{ m}^3/\text{s}$$

$$u = \frac{\pi DN}{60} = \frac{\pi \times 2.5 \times 300}{60} = 39.27 \text{ m/s}$$

## Impulse turbines-Pelton wheel

$$\text{Euler head } H_e = \frac{1}{g} u(V_1 - u) (1 + K \cos \beta'_2)$$

$$= \frac{1}{9.81} \times 39.27 (75.186 - 39.27) (1 + 0.95 \cos 15^\circ)$$

$$= 275.7 \text{ m}$$

$$\text{Hydraulic efficiency } \eta_0 = \frac{H_e}{H} = \frac{275.7}{300} = 0.919$$

$$\text{Overall efficiency } \eta_0 = (\eta_m \eta_h) = 0.919 \times 0.95 = 0.873$$

$$\text{Shaft power } P = \eta_0 \gamma QH = 0.873 \times 9.79 \times 2.362 \times 300 = 6057 \text{ kW}$$

$$\text{Specific speed } N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{6057}}{(300)^{5/4}} = 18.7$$

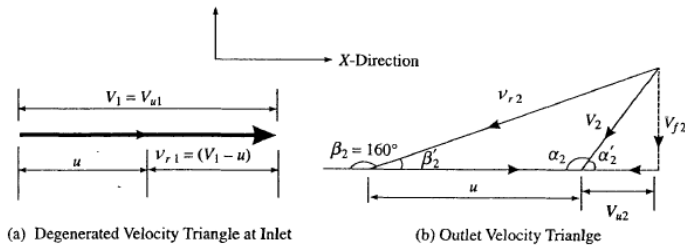
## Impulse turbines-Pelton wheel

### \*\*\*EXAMPLE 4.10

A Pelton turbine has jet velocity of 90 m/s and peripheral velocity of 40 m/s. If the deflection angle of the jet in the bucket is  $160^\circ$ , find the (a) head loss due to bucket friction, and (b) kinetic energy head of exit discharge from the buckets. Take bucket friction coefficient  $K = 0.9$ .

#### Solution

Given:  $V_1 = 90$  m/s,  $u = 40$  m/s,  $\beta_2 = 160^\circ$ ,  $K = 0.9$



(a) Degenerated Velocity Triangle at Inlet

(b) Outlet Velocity Triangle

Fig. 4.20 Velocity triangles, Example 4.10

## Impulse turbines-Pelton wheel

Figure 4.20 shows the inlet and outlet velocity triangles.  $\beta_2' = 180 - 160 = 20^\circ$

Since  $K = 0.9$ , relative velocity  $v_{r2} = 0.9v_{r1} = 0.9(V_1 - u)$ .

$$v_{r2} = 0.9 \times (90 - 40) = 45 \text{ m/s}$$

Let  $\alpha_2'$  be the direction of the absolute velocity  $V_2$  with the peripheral velocity.

$$V_{f2} = V_2 \sin \alpha_2' = v_{r2} \sin \beta_2'$$

$$V_{f2} = 45 \sin 20^\circ = 15.39 \text{ m/s}$$

$$V_{u2} = V_2 \cos \alpha_2' = v_{r2} \cos \beta_2' - u$$

$$V_{u2} = 45 \cos 20^\circ - 40 = 2.29 \text{ m/s}$$

$$\tan \alpha_2' = \frac{V_{f2}}{V_{u2}} = \frac{15.39}{2.29} = 6.72$$

$$\text{Angle } \alpha_2' = 81.537^\circ$$

$$V_2 = \frac{V_{f2}}{\sin \alpha_2'} = \frac{15.39}{\sin 81.537^\circ} = 15.56 \text{ m/s}$$

$$(a) \text{ Energy loss at the bucket} = \frac{v_{r1}^2}{2g} - \frac{v_{r2}^2}{2g} = \frac{(90)^2 - (45)^2}{2 \times 9.81} = 24.2 \text{ m}$$

$$(b) \text{ Kinetic energy head of exit discharge from the buckets} = \frac{V_2^2}{2g} = \frac{(15.56)^2}{2 \times 9.81} = 12.34 \text{ m}$$

**\*\*EXAMPLE 4.19**

The following data pertains to a single-jet Pelton turbine:

Jet diameter = 10 cm	Jet ratio = 12
Coefficient of velocity of the nozzle = 0.98	Speed ratio = 0.46
Bucket friction coefficient = 0.95	Jet deflection angle at the bucket = 165°
Net head = 150 m	

Calculate the torque (a) at the start, and (b) at normal speed.

**Solution**

$D/d = 12$ . Hence,  $D = 12d = 12 \times 0.10 = 1.20$  m

Area of jet  $A_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.10)^2 = 0.007854$  m<sup>2</sup>

Velocity of jet =  $V_1 = C_v \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 150} = 53.16$  m/s

Discharge  $Q = A_1 V_1 = 0.007857 \times 53.16 = 0.4176$  m<sup>3</sup>/s

By Eq. (4.22),

Torque  $T = \frac{\rho Q D}{2} (V_1 - u)(1 + K \cos \beta'_2)$

$\beta'_2 = (180 - \beta_2) = 180 - 165 = 15^\circ$

$T = \frac{0.998 \times 0.4176 \times 1.2}{2} (53.164 - u) [1 + (0.95)(\cos 15^\circ)]$

$T = 0.4794(53.164 - u)$

At start,  $u = 0$  and hence

Torque  $T = 0.4794 (53.164 - 0) = 25.49$  kNm

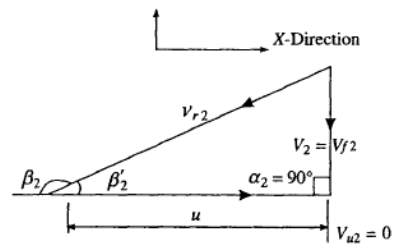
## Impulse turbines-Pelton wheel

**\*\*EXAMPLE 4.26**

For a Pelton turbine the bucket friction coefficient  $K = 0.95$ , coefficient of velocity  $C_v = 0.93$  and the speed ratio  $K_u = 0.46$ . Find the deflection angle  $\beta_2$  of the relative velocity of the jet at the exit that would cause zero velocity of whirl at the exit of the bucket.

**Solution**

Condition for zero velocity of whirl at the exit of the bucket is  $\alpha_2 = 90^\circ$ .



Outlet Velocity Triangle

Fig. 4.24 Velocity triangle, Example 4.26

### Impulse turbines-Pelton wheel

At this condition,  $v_{r2} \cos \beta'_2 = u$  where  $v_{r2} = K(V_1 - u) =$  Relative velocity at the exit of the bucket, (See Fig. 4.24). In this,  $\beta_2 =$  Bucket angle = Deflection angle of the relative velocity of jet at the exit of bucket and  $\beta'_2 = 180^\circ - \beta_2$ .

Hence,  $K(V_1 - u) \cos \beta'_2 = u$

$$\begin{aligned} \cos \beta'_2 &= \frac{u}{v_{r2}} = \frac{u}{K(V_1 - u)} = \frac{1}{K \left( \frac{V_1}{u} - 1 \right)} \\ &= \frac{1}{K \left( \frac{C_v}{K_u} - 1 \right)} = \frac{1}{0.95 \left( \frac{0.98}{0.46} - 1 \right)} = 0.93162 \end{aligned}$$

$$\cos \beta'_2 = 21.31^\circ$$

$$\beta_2 = 180 - \beta'_2 = 158.69^\circ$$